Quantum Computing via Ion Trapping

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Abstract

Quantum computers have the potential to solve complex problems that are infeasible in classical machines. Here, we discuss how quantum computers can be physically implemented using ions contained in a linear trapping device. Furthermore, the challenges and potential for this technology are discussed, all with the perspective of it becoming a staple of quantum computation in the future.

1 Introduction

The mathematical framework for quantum computing has existed since the twentieth century, when it was developed by some of the most notable physicists of the time. However, the most noticeable stimulus for research in quantum computing occurred after both Paul Benioff introduced the quantum-mechanical based Turing Machine [1], and Richard Feynman consequently argued that these systems could be capable of simulating the intrinsic physical properties of nature [2], something classical systems could not achieve. From there, the field saw some noticeable advancements: Peter Shor's prime factorization algorithm [3], Lov Grover's search problem solver [4], and many more.

Nevertheless, this all remains mainly theoretical research since no completely capable quantum computer (QC) has been built. A plethora of ideas have been introduced into how to actually build them, with some of the most prominent being the use of superconducting qubits [5][6] (currently the approach being used by Google [7]), quantum dot-based computers [8] and, finally, the use of cold-trapped ion's internal state as the base for the quantum bits [9]. With the last method, first introduced in 1995 by Cirac and Zoller, being one of the pioneering papers for the physical realization of a QC and will be the focus of discussion throughout this seminar report.

Of course, the fact that so many ideas have been proposed, is nothing but evidence for how relatively young this field is. Anyhow, we are witnessing an increased investment both academically and industrially [10] which reflects the potential this technology has in solving key present day challenges. McKinsey and Company further state that use cases for quantum computing are starting to become realistic for companies in specialized fields [11], which illustrates that a "quantum world" might be closer than we think.

2 Background

2.1 Ions

An ion is an atom or molecule where the electronic structure is not identically distributed between protons and electrons. Specifically, by gaining or losing an electron, the particle gains a net charge in a process called ionization.

In particular, by shining a laser with sufficient energy on an atom, we can effectively force it to drop a specific number of electrons thus becoming an ion. This process is called ionization and the energy required for the process is known for a specific atom/number of electrons pair.

2.2 Ion traps

First, we need to isolate the ion such that other particles do not interfere with the very delicate conditions for ion trapping. This is, naturally, done via vacuum chambers encasing the trapping device. Any interaction with the system is, therefore, accomplished with optical and electrical signals.

Furthermore, we should consider that the Coulomb force (1) is in effect here: particles with differently-signed charges will attract each other, while equally-signed particles will be mutually repelled [12].

$$F_1 = \frac{q_1 q_2}{4\pi\epsilon_0} \frac{r_1 - r_2}{|r_1 - r_2|^3} \tag{1}$$

From there, one might hope that encasing the particle with positively charged electrodes (assuming a positively charged ion) will confine it indefinitely. However, due to Earnshaw's theorem [13], we know it is impossible to contain a stationary particle in equilibrium resourcing only to the electrostatic interaction of charges.

Therefore, for this particular problem, we should resource to a dynamical electrical field.

2.2.1 Traditional Paul traps

In 1990, German physicist Wolfgang Paul, proposed a device with a similar arrangement [14], but that repeatedly alternates between electrical field configurations to, successfully, confine the ions.

Suppose we continue to surround the particles with electrodes in all axes. A linear Paul trap fixes one axis with a positive voltage. This is done via two cone-shaped electrodes, named Endcap electrodes. Surrounding the particle in the other axes are rectangular-shaped electrodes named Blade electrodes. Each pair contains an opposite charge so that the ion is always attracted to two of the electrodes.

In static conditions, this creates an electrical field with a saddle shape, where the particle is drawn to the negative charge. However, if we rotate the electric field with sufficient frequency, such that the rotation is faster than the attraction of the particle, we can effectively, and periodically, draw the particle to different blades such that it remains confined in a sphere of arbitrary radius.

In practice, the rotation of the electrical field could be done by physically rotating the device, but it is much easier to apply differently signed voltages to each pair of blade electrodes in a oscillating fashion. Effectively rotating the electrical field with a certain frequency.

Furthermore, given that a collection of ions also suffer from Coulomb's law, it is possible to engineer a trapping device that is able to confine them simultaneously [14].

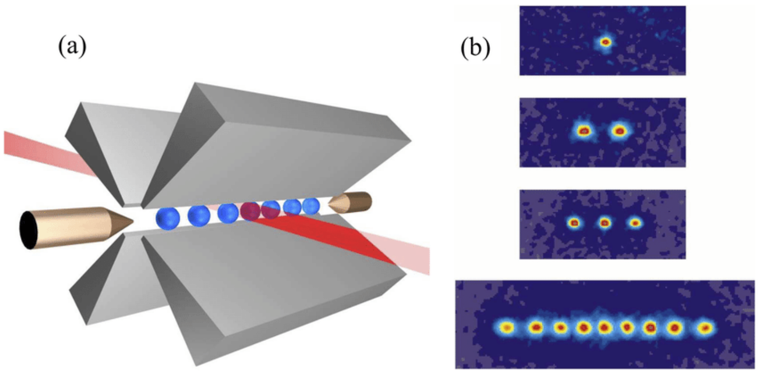

Figure 1: a) The traditional linear Paul trap, composed of four blade electrodes and two endcap electrodes confining a string of ions. A laser beam shines on an ion illustrating the application of a single qubit gate. b) Photographs of actual ions confined in a Paul trap. Adapted from [15].

2.2.2 Planar Paul traps

It seems obvious to us, a generation with classical processors in the nanometer scale, that any device that requires scale and engineering precision would benefit from microfabrication processes. Of course, this applies to quantum processors as well. The description for transitioning from the traditional Paul trap to a microchip has been published since around 2006 [16], and the field has seen some remarkable advancements since then. Namely, in terms of modularity of the chip and the capability of moving ions around [17].

However, some new challenges arise when this manufacturing scale is considered, especially regarding the addressing and heating of individual trapped ions [19].

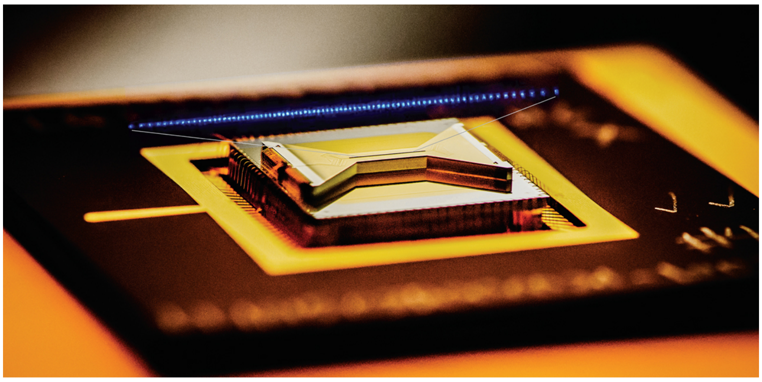

Figure 2: An example of an ion trap based on a semiconductor chip. The ions sitting above it are a zoomed in picture of 80 $^{171}Yb^{+}$ ions glowing. Adapted from [18].

3 How do we build a quantum computer?

When we discuss what aspects are necessary to construct a Quantum Computer, we have to refer to DiVicenzo's criteria [20]. He states that for any physical implementation to be successful, it must fulfill five (plus two) criteria:

- Scalability with well-defined qubits
- Initializing qubits to a simple fiducial state
- Long relevant decoherence times
- A "universal" set of quantum gates
- A qubit specific measurement capability

The last two criteria are: interconverting stationary and flying qubits and faithfully transmitting flying qubits between specified locations. However, these only apply when quantum communication is being considered and, since a physical implementation of QC is a landmark achievement by itself, those are not going to be discussed in this report.

3.1 Qubits

Taking into account that atoms have naturally occurring vibrations that differ according to their energy level, it's only natural that we can leverage them to form the quantum states. Cirac and Zoller proposed using a particle's energy ground state as $|0\rangle$ and it's excited state as $|1\rangle$. Therefore, when we dwell upon the string of N ions present in a ion trap at a particular time, we find a macroscopic superposition of possible states (2).

$$|\Psi\rangle = \sum_{x=0}^{2^N-1} c_x |x\rangle \tag{2}$$

Each possible state is associated with a coefficient c_x akin to the probability of the QC being in the specific state. Moreover, x corresponds to the binary decomposition of a particular number and, since each ion can be targeted with lasers, the preparation and readout of states can be done in a simple manner as we'll see in a further section.

Furthermore, in this modality of quantum computers, we can either have optical or hyperfine qubits. They differ only in what is considered the excited state: for the optical qubit, $|1\rangle$ is a metastable excited state; whereas, in hyperfine qubits, $|1\rangle$ is the first excited level above the ground state.

3.2 Fiducial state

In layman's terms, this means that the state of the QC must be known before computation begins, such that it's results are trustworthy. Keeping in mind that every state can be obtained starting from an initial, low energy, $|0\rangle$ state [21]. This equates to the most common approach in initializing the quantum registers. From there, physicists prepare the desired state by applying a sequence of quantum gates as needed.

Now, we should consider how to actually prepare the initial state. As discussed before, we regard $|0\rangle$ as the ground energy level in a particular atom. Moreover, the overall string of ions confined has to be set to it's lowest energy level, this ensures the computation results are trustworthy as it is the base for the Cirac-Zoller gate we will soon introduce.

The ground state is achieved with extreme cooling procedures, namely, the one mentioned in the original article, sideband cooling [22]. Essentially, when a laser is tuned specifically to the red sideband frequency of a particular atom, it inflicts a transition from a lower energy level to an excited state, simultaneously reducing the motional state of the atom by one discrete level. However, the energy absorbed is subsequently released in the form of a photon, resulting in the net effect of lowering its energy and motional level to the lowest possible discrete state with high probability if the procedure is repeated to a sufficient degree [23].

In other words, with this method it is possible to reliably prepare a complete $|0\rangle$ state, both in terms of the electronic state of every individual ion and the overall motional level of the oscillating system.

3.3 Decoherence time

One of the main benefits in building quantum computers with ion-trapping technology is that the decoherence time of states is relatively high when compared to other solutions. With this method, decoherence between the expected state of a qubit and its actual state occurs due to the interaction of the ions with the surrounding environment, since collisions between atoms and interactions with naturally occurring electric fields are bound to happen. Furthermore, we should also mention the spontaneous emission of energy in excited particles, leading to the decay of a particle into a lower energy level.

However, there are techniques, and evidence to support they work, that suppress most of these concerns. Regarding the storage of the delicate quantum information, by encasing the system in a vacuum chamber, we remove most of the possibilities for the system to interact with neighboring particles and electrical fields, thus preserving its intrinsic structure.

On the other hand, spontaneous emission of energy is mainly a non-factor in some specific ions, since the excited state used for $|1\rangle$ is relatively stable and is able to stay intact for the lifetime of most computations [24]. In particular, in 1997, for the $^{171}Yb^{+}$ ion, coherence times of more than 10 minutes have been measured [25].

3.4 Quantum Gates

Initially, we should start by addressing the fact that any gate can be constructed with single-qubit gates and an arbitrary 2-qubit gate [26], thus forming a universal set of quantum gates. Therefore, a natural limitation in how to build a QC is how to achieve a working 2-qubit gate, something that requires "communication" between two particles.

3.4.1 Single qubit gates

Consider the interpretation of single qubit gates as rotations on the block sphere. We know from the Z-Y decomposition that an arbitrary 2x2 matrix can be decomposed into R_z and R_y rotations with specific real valued angles [21]. Furthermore, since each ion is addressable with correctly tuned lasers, we'll discuss that any rotation can be performed and, therefore, the transition from a electronic ground state to each possible state is theoretically possible [21].

Specifically, the tuning of the lasers depends on the ion in question. For instance, the use of a resonant frequency should oscillate between the two electronic states being used as $|0\rangle$ and $|1\rangle$ [27]. Going even further, since an atom makes up for a two level system and, when subject to an oscillatory driving field such as a laser, it oscillates between its states according to a specific innate frequency called the Rabi frequency [28].

On the other hand, tuning the pulse length and phase results in a rotation of arbitrary accuracy [27], as described in (3). Making it that a rotation on the Bloch sphere is akin to subjecting a ion to a particular laser configuration.

$$R(\theta,\phi) = \exp(i\theta/2(e^{i\phi}\sigma_+ + e^{-i\phi}\sigma_-)) \tag{3}$$

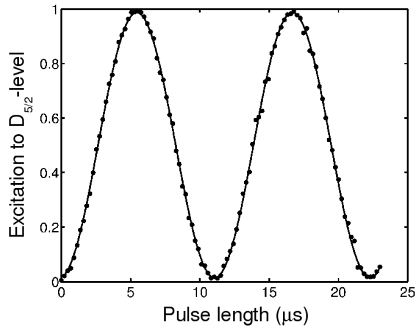

Figure 3: Carrier Rabi Oscillations of a $^{40}Ca^{+}$ ion. Each sample is the average measurement after 1000 experiments, given a specific amount of time where the ion was subjected to a laser. Adapted from [27].

Interpreting figure 4: in a), a pulse of length π and 0 relative phase to the Rabi frequency shines upon the atom, this equates to a rotation around the x-axis on the Bloch sphere according to the right-hand rule; when the phase is switched by $\pi/2$, we can interpret this as a rotation around the y axis; therefore, when projecting on the z axis for measurement, the switching of phase has no visible effect.

Regarding b), something very similar occurs, but the switching of phase happens when the atom is in a superposition state. Specifically, the atom is positioned at the intersection of the y-axis and the Bloch sphere, therefore, a rotation around y does nothing. With that in mind, during this period, a projection of the atom on the z axis appears to be constant relative to time.

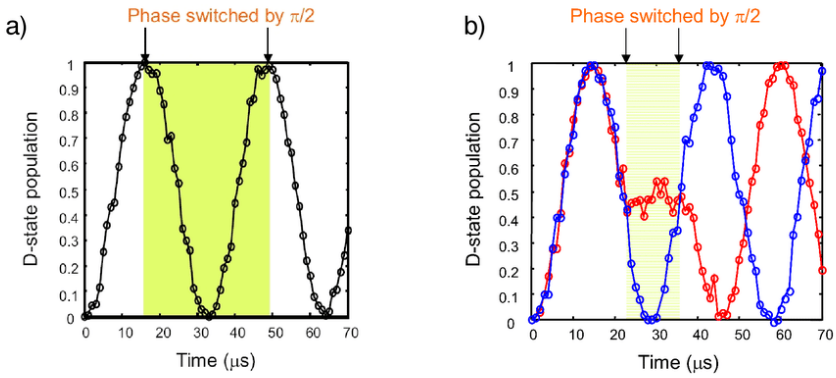

Figure 4: Illustration of single qubit rotations. a) shows the measurements during the pulse sequence $R(\pi, 0)R(2\pi, \pi/2)R(\pi, 0)$. b) shows the measurements for the pulse sequence $R(3\pi/2, 0)R(\pi, \pi/2)R(3\pi/2, 0)$ in red and the regular Rabi oscillations in blue. Adapted from [27].

Moreover, the physical implementation of single qubit gates in ion traps has been shown to be accurate up to a certain degree [29], but the accuracy threshold for reliable quantum computing has not yet been achieved [30]. More specifically, the rate at which errors occur when we perform a specific gate has to be extremely smaller than the rate at which computations are performed.

3.4.2 Cirac-Zoller CNOT gate

The key idea introduced in the original and pioneering article. Foremost, note that a cNOT gate can be achieved with the Hadamard and Controlled-Z gates [21]. Therefore, since we've discussed how the Hadamard is implemented as a single-qubit gate, the novel idea is how a Controlled-Z gate works in practice.

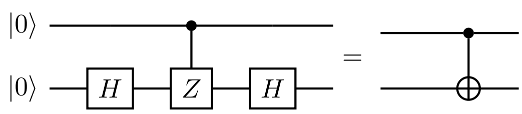

Figure 5: Constructing a cNOT gate from the Controlled-Z and Hadamard gates

To start with, we'll remind the reader that the ions confined in the trapping device suffer from Coulomb's Force and therefore are equidistant from each other along one of the axis. Moreover, since they've been sufficiently cooled per assumption, a laser targeting a particular ion can only affect the atom's internal energy state and the overall chain's motion [31]. The latter is accomplished by detuning the laser from the internal transition by a predetermined amount of energy.

Additionally, since the chain's center of mass (CM) motion is induced from the control ion's quantum information, we can consider this particular motion of the ion chain an auxiliary quantum state. In fact, the motion of the chain has several possible states. However, one of the preconditions for this experiment is achieving the Lamb-Dicke regime, in which transitions larger than one quantum of motion are generally suppressed.

With these two ingredients, the first step in the Controlled-Z gate is the transferring of information from the control qubit to the chain's CM motion. As mentioned before we can force the transition $|e\rangle|0\rangle \longrightarrow |g\rangle|1\rangle$ with a proper polarization of the laser field and, in fact, this suffices to complete the information transfer, see (4).

$$(a|g\rangle + b|e\rangle)|0\rangle = a|g\rangle|0\rangle + b|e\rangle|0\rangle \longrightarrow a|g\rangle|0\rangle + b|g\rangle|1\rangle = |g\rangle(a|0\rangle + b|1\rangle) \tag{4}$$

Lastly, we can choose a laser polarization such that it drives a transition on the target ion only if the auxiliary state is $|1\rangle$. Therefore making it possible to establish communication between two separate atoms through the chain's CM motion.

Essentially, putting everything together: a laser targets ion m, transferring its internal energy state to the motion of the chain; from there, a second laser targeting ion n only has an effect if the chain's state is $|1\rangle$; finally, a similarly tuned laser as in the first step is shined upon the first ion, restoring both it and the overall chain's original states, while simultaneously inducing a -1 phase on ion n, depending on the original state of ion m. Effectively, completing a Controlled-Z gate. The overall unitary operator for this experiment is (5) as per the original paper [9], which delves into more detail on how it should be derived.

$$\hat{U}_m^{k,q}(\phi) = \exp[-ik\frac{\pi}{2}(|e_q\rangle_n \langle g| a e^{-i\phi} + H.c.)] \tag{5}$$

Furthermore, we can observe the evolution of the system in (6) exhausting the possible configurations it may have.

$$\begin{array}{rrrr}
\hat{U}_m^{1,0} & \hat{U}_n^{2,1} & \hat{U}_m^{1,0} & \\
|g\rangle_m |g\rangle_n |0\rangle \longrightarrow & |g\rangle_m |g\rangle_n |0\rangle \longrightarrow & |g\rangle_m |g\rangle_n |0\rangle \longrightarrow & |g\rangle_m |g\rangle_n |0\rangle \\
|g\rangle_m |e_0\rangle_n |0\rangle \longrightarrow & |g\rangle_m |e_0\rangle_n |0\rangle \longrightarrow & |g\rangle_m |e_0\rangle_n |0\rangle \longrightarrow & |g\rangle_m |e_0\rangle_n |0\rangle \\
|e_0\rangle_m |g\rangle_n |0\rangle \longrightarrow & -i|g\rangle_m |g\rangle_n |1\rangle \longrightarrow & i|g\rangle_m |g\rangle_n |1\rangle \longrightarrow & |e_0\rangle_m |g\rangle_n |0\rangle \\
|e_0\rangle_m |e_0\rangle_n |0\rangle \longrightarrow & -i|g\rangle_m |e_0\rangle_n |1\rangle \longrightarrow & -i|g\rangle_m |e_0\rangle_n |1\rangle \longrightarrow & -|e_0\rangle_m |e_0\rangle_n |0\rangle
\end{array} \tag{6}$$

As a last note, the first successful realization of the Cirac-Zoller gate was completed in 2003 [32], eight years after the publishing of the original paper, marking a first step in making quantum computers more than a theoretical concept.

3.5 Measuring

As for the measurement of the quantum state after computations have been performed, we'll rely on the observation of quantum jumps between internal energy states [33]. Note that for pedagogical purposes, we will focus on optical qubits that rely on relatively larger energy transitions. Hyperfine qubits are measured with the same principle, but leverage the concept of shelving to project a ground state in to a meta-stable state (D).

Notice on figure 6 how a transition is driven with a laser pulse from the ground state to the $D^{5/2}$ state (unlabeled) or, in other words, a transition from the $|0\rangle$ to the $|1\rangle$ state in optical qubits. Hyperfine qubits utilize the first energy surplus state as $|1\rangle$. Furthermore, this forced transition is the only path for the transition $S^{1/2} \leftrightarrow D^{5/2}$ to occur.

Consider that the P states are excited states, therefore when they naturally decay to the ground state they release photons with a very high frequency, making the ion fluorescent. However, in D states the same doesn't occur since the transition from the P state takes a relatively long time and the photons are released in a sporadic fashion.

With all this in mind, upon shining a laser set up to drive the transition S $\leftrightarrow$ P on our ion, if we are equipped with a photon detector as depicted in 7, we can detect its state with high accuracy by capturing the frequency of emitted photons.

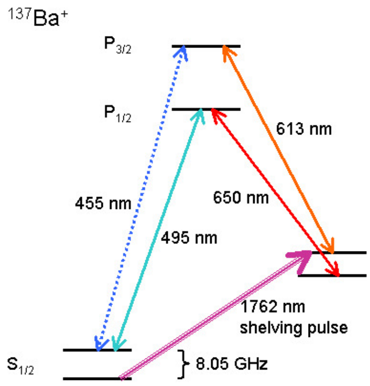

Figure 6: Valid transitions between internal energy states in a Barium ion [34]

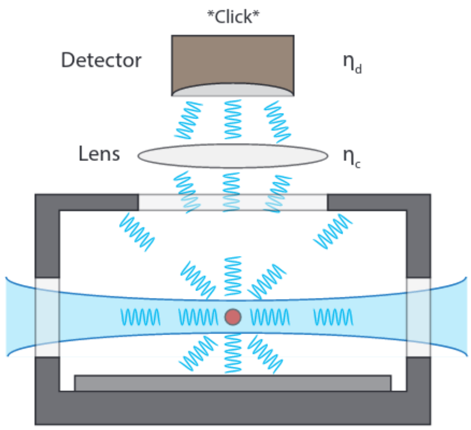

Figure 7: Schematic of a device capable of readout on trapped ions. A laser shines on the ion, causing it to scatter photons which are registered on a photon detector. Adapted from [35].

On a practical notice, using quantum jumps sprinkled with other techniques, such as tracking photon arrival times, has achieved a precision of circa 99.9% [36]. Making qubit readout extremely accurate and in the order of hundreds of microseconds.

4 Limitations and potential

Concluding, now that all the steps required to build a QC with trapped ions have been explained, some considerations about their pros and cons are required.

First of all, we should be aware that fidelity and accuracy in this technology is considered relatively high, or similar, when compared to other methods of quantum computing. For instance, state coherence lifespan is in the order of minutes, much longer than the time it takes to perform computations.

Another aspect is the accuracy that quantum gates sustain, we've discussed that the precision for single qubit gates lies in the 99.9 percentile. Meanwhile, 2-qubit gates share the same accuracy for hyperfine qubits, or 99.6% for optical qubits. Both, extremely acceptable in the optics of quantum error correction. Moreover, qubit initialization and readout are also straightforward given the current research available. Both, having high precision techniques available, displaying impressive 99.9% or higher percentiles.

In contrast, one disadvantage of ion trapped quantum computing may very well be the slow (relatively) time it takes to perform 2-qubit gates. Compared to other methods, like superconducting qubits who have measured times in the order of nanoseconds [37], ion QCs lag behind, performing logic gates in the magnitude of microseconds, specifically $2.6\mu s$ as per Schäfer et al. [38].

Finally, we should also consider that this technology is harder to scale then competing modalities of quantum computing. While the trapping of ions isn't necessarily a concern, the fact that they need to be addressable poses a serious problem. Currently, we have an 127-qubit processor made by IBM commercially available and with a rapidly increasing qubit count in their roadmap [39]. On the other hand, one of the most successful companies behind ion trapped quantum computers, IonQ, claims to have properly performed single-qubit gates on chains of 79 ions, and 2-qubit gates on only 11 ion pairs [40].

Concluding, at this point ion trapping based quantum computers can be seen as reliable with high fidelity and accuracy, but with scaling issues in terms of qubits and gate computation speed. With that in mind, the competing technologies also have their problems and the optimal medium for quantum computing hasn't come to a consensus. However, at this point, it is remarkable enough that we have come to the point where these technologies are becoming enterprise worthy and not a long, and distant, dream.

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